Bachelor's Thesis

Bachelor's Degree in Industrial Technology Engineering

Interactive Tools to learn Automatic Control for Mobile Devices

MEMORY

Jingrui Zhang
Ramon Costa Castelló

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ETSEIB

Escola Tècnica Superior d'Enginyeria Industrial de Barcelona

UPC

Abstract

The current Bachelor's thesis's objective is to create virtual laboratories using the software tool Easy Javascript Simulation; namely two tanks system and Pendulum. These simulations created using Easy Javascript Simulation can be executed with web browser and also with mobile devices such as smartphones or tables and can be used in education.

The accomplishment of the development of these simulations involved the study of a nonlinear system, the state space analysis and, linearization of a nonlinear system, state feedback study and state observer study.

The present paper is organized as follow. First of all there is an introduction of Easy Javascript Simulation. Then, the state space analysis is presented, including the study of equilibrium points, linearization of a nonlinear system, the state feedback analysis and also the state observer analysis. Following these there is the study of the two tanks system and Pendulum system, involving the instruction of each application and some examples done using these applications.

Finally, there is also the realization of the project budget and environmental impact at the end of this project.

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1. Introduction

Nowadays, interactivity plays an extremely important role in automatic control. The reason is that almost all the ideas and concepts of automatic control can be represented by graphics. Moreover, all these concepts are captured and understood better via visualization. Computer is a really useful tool in education in the sense, the reason is that it can provide the visualization and manipulation of these automatic control's ideas in an interactive way.

Since mobile devices are more and more being used in present society, with the tendency to replace the computer for people's everyday life because of the easy access to the Internet and its ability to provide a good communication and interaction as computer does. The creation of simulations for pedagogical use for mobile devices is such an important issue nowadays.

1.1 Objectives

The current research is focused on the creation of two interactive virtual laboratories for non-linear control system with the Easy Javascript Simulations; these virtual laboratories are two tanks system and pendulum. Both of these simulations created have to be highly interactive so that they can be used to support in teaching process. In addition, these simulations can be executed not only with web browser but also among mobile devices (smartphones or tablets) as an applet.

2. Easy Javascript Simulation, EjsS

2.1 Description

Easy Javascript Simulation, also called as EjsS, is a software tool designed by *Francisco Esquembre* in the Open Source Physics project. The main intention of creating EjsS is to help teachers, science students or researchers to create scientific simulations. This software tool provides Javascript as the programming language. The simulations created using this software tool can be executed with WWW (*World Wide Web*) and also among mobile devices.

As the author *Francisco Esquembre* said, EjsS is designed for science teachers and students, who are more interested in the simulated phenomenon instated of the technical aspects of programming. EjsS provides several simplified tools to its user to create these simulations. Therefore, the user can dedicate their time on the scientific aspects of their simulation. However, the result generated automatically by EjsS is as efficient as a program creates by a professional. [1]

To design a simulation, EjsS provides three work panels. These workpanels are namely *Description*, *Model* and *HtmlView*. The figure below shows an illustration about three work panels mentioned.

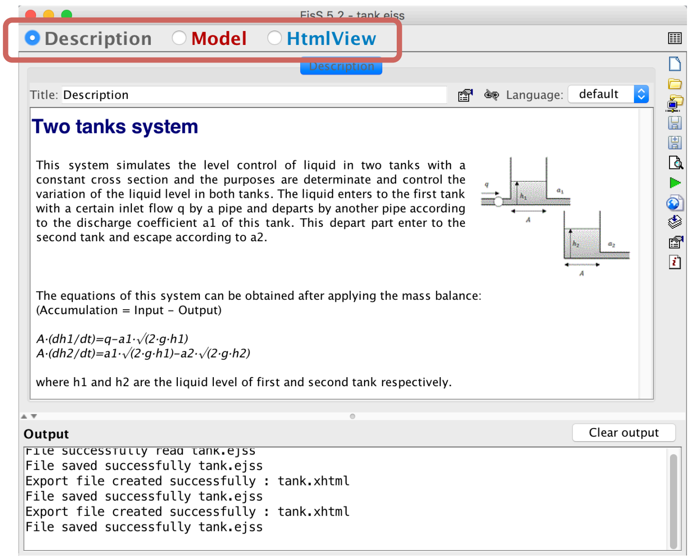

Figure 1: Easy Javascript Simulation's first work panel

The first work panel (Figure 1), *Description*, permit users to create and edit the description of the simulation. This description can be some activities related to the simulation created or just an introduction of the simulated system. The second work panel, Model, is dedicated to create variables, which can be initialized (*Initialization*) and parameters to define a system (the *variable* panel), this panel also allows its user to define the algorism that describes the movement of the model (*Evolution)*; user can define the evolution of the system by first order Ordinary Differential Equation or using functions. Moreover, this second work panel can be used to define relations among variables in necessary case (*Fixed relations*) and possible Javascript function (*Custom*). In this present project, this Custom panel is used to settle the functions of equilibrium points. Finally, the access to external Javascript libraries (*Elements*) is provided too (Figure 2). The last work panel, *HtmlView*, shown in figure 3, is for creating graphical user interface, visualization, user interaction, program control and display output for the simulation. [2]

Figure 2: The second work panel: Model

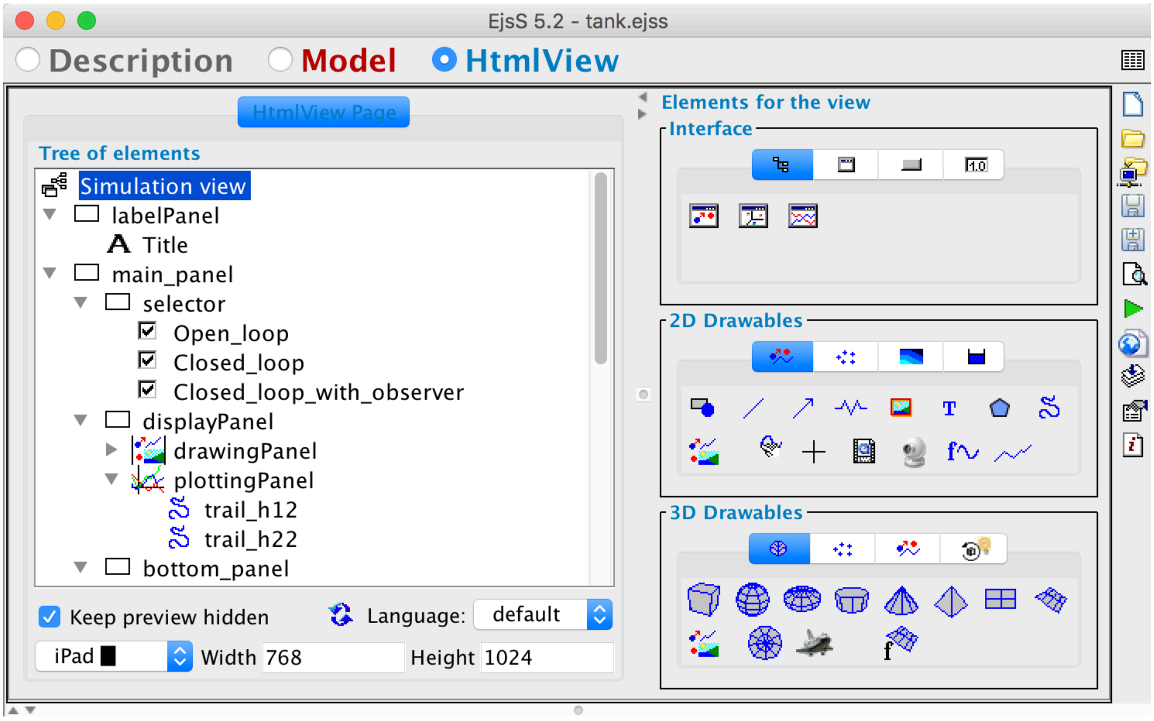

Figure 3: The third work panel: HtmlView

2.2 Interactivity

With the virtual laboratories created in the current project, the users are expected to realize the following tasks related to a nonlinear system:

- Visualize the temporal variation of state variables in each system.
- Modify initial conditions of state variables.
- Obtain equilibrium points of both systems.
- Realize the state feedback study; the parameters of state feedback gain are editable.
- Do the state observers study; the values of state observer gain can be modified.

3. State space analysis

In recent years, engineering systems tend to be more complicated because of the complex task and exactitude it requires. A complex system can present more than one inputs and outputs, can be linear or nonlinear; moreover, it can also be time varying or time invariant system. To reduce the complexity and the requirement of easy access to computers, modern control theory has been introduced since sixtieth century; it is focused on the concept of state and time-domain approach.

First of all, the definitions of state, state variables, state space equations are introduced.

A ***state*** of a dynamic system can be characterized with the minimum numbers of variables at a determinate time t_0, with all the inputs for any time greater than or equal to the given time $(t \geq t_0)$ can define the evolution of this system in any time $t \geq t_0$.

The ***state variables*** are the minimum amount of necessaries variables to describe the behaviour of a dynamic system at any time *t*. These variables must be linearly independent. It is important to take into account that for these state variables, the properties of physically measurable and observable are highly recommended in practice even though they are not necessary; the reason is because of the requirement of the feedback of the optimal control law for all the state variables.

The ***state equations*** describe the behaviour of system with the representation of the relations between input variables, output variables and the state variables. The following paragraphs describe an example for the state space analysis of a time varying system. [3]

Supposing a system with multiple inputs and outputs, where the inputs are defined by $u_1(t), u_2(t), \dots, u_r(t)$ and the outputs are $y_1(t), y_2(t), \dots, y_m(t)$. Supposing n state variables $x_1(t),\ x_2(t), \dots, x_n(t)$. The block diagram of this given system is:

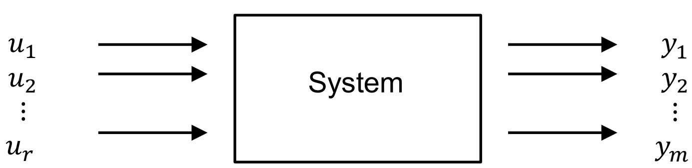

Figure 4: Block diagram of a several inputs and outputs system

The system defined previously can be described by the state equations:

$$
\begin{aligned}
\dot{x_1}(t) &= f_1(x_1, x_2, \cdots, x_n; u_1, u_2, \cdots, u_n; t);\\
\dot{x_2}(t) &= f_2(x_1, x_2, \cdots, x_n; u_1, u_2, \cdots, u_n; t);\\
&\vdots\\
\dot{x_3}(t) &= f_3(x_1, x_2, \cdots, x_n; u_1, u_2, \cdots, u_n; t);\\
\\
y_1(t) &= g_1(x_1, x_2, \dots, x_n; u_1, u_2, \dots, u_r; t);\\
y_2(t) &= g_2(x_1, x_2, \dots, x_n; u_1, u_2, \dots, u_r; t);\\
&\vdots\\
y_m(t) &= g_m(x_1, x_2, \dots, x_n; u_1, u_2, \dots, u_r; t)
\end{aligned}
\tag{1}
$$

Therefore, if we define:

$$
\boldsymbol{u}(t) = \begin{bmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_r(t) \end{bmatrix}, \qquad \boldsymbol{y}(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_m(t) \end{bmatrix}, \qquad \boldsymbol{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix}
$$

$$
\boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}, t) = \begin{bmatrix} f_1(x_1, x_2, \dots, x_n; u_1, u_2, \dots, u_r; t) \\ f_2(x_1, x_2, \dots, x_n; u_1, u_2, \dots, u_r; t) \\ \vdots \\ f_3(x_1, x_2, \dots, x_n; u_1, u_2, \dots, u_r; t) \end{bmatrix},
\tag{2}
$$

$$
\boldsymbol{g}(\boldsymbol{x}, \boldsymbol{u}, t) = \begin{bmatrix} g_1(x_1, x_2, \dots, x_n; u_1, u_2, \dots, u_r; t) \\ g_2(x_1, x_2, \dots, x_n; u_1, u_2, \dots, u_r; t) \\ \vdots \\ g_m(x_1, x_2, \dots, x_n; u_1, u_2, \dots, u_r; t) \end{bmatrix}
$$

The state equations of the given system became:

$$
\begin{aligned}
\dot{x}(t) &= \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}, t)\\
\\
y(t) &= \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{u}, t)
\end{aligned}
\tag{3}
$$

3.1 Equilibrium point

The concept of the equilibrium points is really important to the state equation. An equilibrium point *x*=*x** is defined as a point in a system where the temporal variation of the state equations of the given system evaluated in this point does not exist. In other words, equilibrium points can arrest the movement of a system. Furthermore, equilibrium points can be calculated by equalizing the state equations to zero. In general, a nonlinear system has several equilibrium points. [4]

Considering the preview example, where the system is described by the following state equation:

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{u}, t) \\ \boldsymbol{y}(t) &= \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{u}, t) \end{aligned} \tag{4}$$

The equilibrium points are calculated by:

$$\begin{aligned} 0 &= \boldsymbol{f}(\boldsymbol{x}^*, \boldsymbol{u}^*, t) \\ \boldsymbol{y}(t) &= \boldsymbol{g}(\boldsymbol{x}^*, \boldsymbol{u}^*, t) \end{aligned} \tag{5}$$

where $\boldsymbol{x}^*$ and $\boldsymbol{u}^*$ are the equilibrium points of this system.

As explained before, these equilibrium points $\boldsymbol{x}^*$ and $\boldsymbol{u}^*$ have the property to arrest the movement of the given system and maintain the state $\boldsymbol{x}^*, \boldsymbol{u}^*$ in any future time.

3.2 Linearization of a nonlinear system around an equilibrium point

It is possible to obtain a linear approximation of a nonlinear system around an equilibrium point applying the Taylor expansion close to this equilibrium point. Considering a nonlinear system defined by the functions $\boldsymbol{f}$ and $\boldsymbol{g}$[1], the expressions of the Taylor expansion around the equilibrium point of this system are:

[1] The functions $\boldsymbol{f}$ and $\boldsymbol{g}$ are the same as the example used at the beginning of this chapter but without the temporal dependence.

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x},\boldsymbol{u}) \approx \boldsymbol{f}(\boldsymbol{x}^*,\boldsymbol{u}^*) + \left[\left.\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{x}}\right|_{\boldsymbol{x}^*,\,\boldsymbol{u}^*} (\boldsymbol{x}-\boldsymbol{x}^*) + \left.\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{u}}\right|_{\boldsymbol{x}^*,\,\boldsymbol{u}^*} (\boldsymbol{u}-\boldsymbol{u}^*) \right] + \cdots$$

$$\boldsymbol{y} = \boldsymbol{g}(\boldsymbol{x},\boldsymbol{u}) \approx \boldsymbol{g}(\boldsymbol{x}^*,\boldsymbol{u}^*) + \left[\left.\frac{\partial \boldsymbol{g}}{\partial \boldsymbol{x}}\right|_{\boldsymbol{x}^*,\,\boldsymbol{u}^*} (\boldsymbol{x}-\boldsymbol{x}^*) + \left.\frac{\partial \boldsymbol{g}}{\partial \boldsymbol{u}}\right|_{\boldsymbol{x}^*,\,\boldsymbol{u}^*} (\boldsymbol{u}-\boldsymbol{u}^*) \right] + \cdots \quad (6)$$

To achieve the linearization form of a nonlinear system around the equilibrium point is necessary to determine some new variables, which are defined as the displacement respect the respective equilibrium points. These new variables are calculated by the subtractions below:

$$\chi = \boldsymbol{x} - \boldsymbol{x}^*$$

$$\gamma = \boldsymbol{y} - \boldsymbol{y}^* \quad (7)$$

$$\boldsymbol{\mu} = \boldsymbol{u} - \boldsymbol{u}^*$$

Apart of the new variables defined, it is also more convenient to nominate the *Jacobian matrix* of the functions $\boldsymbol{f}(\boldsymbol{x},\boldsymbol{u})$ and $\boldsymbol{g}(\boldsymbol{x},\boldsymbol{u})$:

$$\boldsymbol{A} = \left.\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{x}}\right|_{\boldsymbol{x}^*,\,\boldsymbol{u}^*} \qquad \boldsymbol{B} = \left.\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{u}}\right|_{\boldsymbol{x}^*,\,\boldsymbol{u}^*}$$

$$\boldsymbol{C} = \left.\frac{\partial \boldsymbol{g}}{\partial \boldsymbol{x}}\right|_{\boldsymbol{x}^*,\,\boldsymbol{u}^*} \qquad \boldsymbol{D} = \left.\frac{\partial \boldsymbol{g}}{\partial \boldsymbol{u}}\right|_{\boldsymbol{x}^*,\,\boldsymbol{u}^*} \quad (8)$$

where matrix **A** is called the state matrix, **B** is the input matrix, **C** the output matrix and **D** the direct transmission matrix.

Finally, the Taylor expressions around the equilibrium point, or the linearization form of a nonlinear system around the equilibrium point are:

$$\dot{\chi} \approx \boldsymbol{A}\cdot\chi + \boldsymbol{B}\cdot\boldsymbol{\mu}$$

$$\gamma \approx \boldsymbol{C}\cdot\chi + \boldsymbol{D}\cdot\boldsymbol{\mu} \quad (9)$$

3.3 State feedback system

The local stabilization of a nonlinear system around the equilibrium points can be achieved by adding a control law. The most common control law used is $u = -\boldsymbol{K} \cdot \boldsymbol{x} + v$, where $\boldsymbol{x}$ is the vector of state variable, $\boldsymbol{K}$ is an one row matrix and it is called the state feedback gain matrix, and the v is an auxiliary input. In current project, this auxiliary input is a null input because all the systems studied are nonlinear and the linearization of these nonlinear systems are done around equilibrium points; then the control law became $u = -\boldsymbol{K} \cdot \boldsymbol{x}$. The assumption of all state variables is physically measurable for feedback is needed. The following figure presents a block diagram of a nonlinear system after adding a controller.

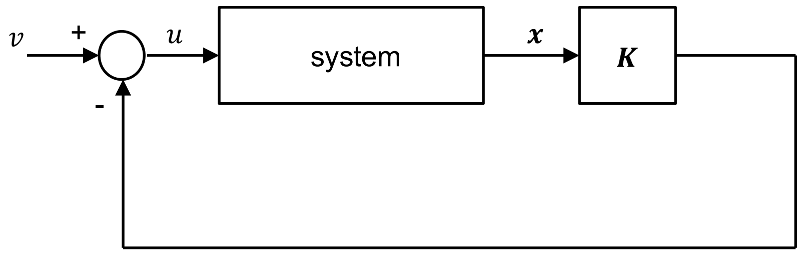

Figure 5: Closed loop control system with controller

Bearing in mind that the state equation of a nonlinear system can be written as:

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{b}\boldsymbol{u} \tag{10}$$

With the control low $u = -\boldsymbol{K} \cdot \boldsymbol{x}$ this equation can be written as:

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{b} \cdot (-\boldsymbol{K} \cdot \boldsymbol{x}) = [\boldsymbol{A} - \boldsymbol{b}\boldsymbol{K}]\boldsymbol{x} \tag{11}$$

This method of stabilization of a nonlinear system can only be applied if the system is controllable[1], and the closed loop stability depends on the eigenvalues of $\boldsymbol{A} - \boldsymbol{b}\boldsymbol{K}$.

[1] Controllability: a system is controllable if the controllability matrix has rank *n*. [3]

The controllability matrix is $[B \quad AB \quad \cdots \quad A^{n-1}B]$.

3.4 State observer system

Is necessary estimate the state variable in case if this variable is not physically measurable for feedback. And the estimation of unavailable state variable is called observation. State observers can be applied if and only if the system is observable[1]. The mathematical model of the observer is:

$$\dot{\tilde{\boldsymbol{x}}} = \boldsymbol{A}\tilde{\boldsymbol{x}} + \boldsymbol{B}\boldsymbol{u} + \boldsymbol{L}(\boldsymbol{y} - \boldsymbol{C}\tilde{\boldsymbol{x}}) \tag{12}$$

where $\tilde{x}$ is the estimated state, $\boldsymbol{L}$ is an one column matrix and it is denominated as the observer gain matrix, $\boldsymbol{C}\tilde{\boldsymbol{x}}$ is the estimated output and $\boldsymbol{L}(\boldsymbol{y} - \boldsymbol{C}\tilde{\boldsymbol{x}})$ is the term of the output prediction error.

[1] Observability: a system is observable if the observability matrix has rank *n*. [3]

The controllability matrix is $\begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}$.

4. Two tanks system

4.1 Description

The two tanks system simulates the level control of liquid in two tanks with a constant cross section and the purposes are determinate and control the variation of the liquid level in both tanks. The current study of two tanks system involves three modes, the open-loop mode, the closed-loop mode, corresponding to the state feedback system and the closed loop mode with observers, which is the state observer system.

4.2 Study of no lineal equations

As mentioned above, the present study of two tanks system involves three modes. The figure below illustrates the behaviour of the open loop mode; the liquid enters to the first tank with a certain value of inlet flow q by a pipe and departs by another pipe according to the discharge coefficient of this first tank a_1. This depart part of liquid enters to the second tank and escapes according to a_2 (discharge coefficient of the second tank). This is how the variation of the liquid level in both tanks takes place. In closed loop mode, there will be a feedback system of the output of both tanks whereas in closed loop mode with observers, the feedback system is only applied in the output of the second tank.

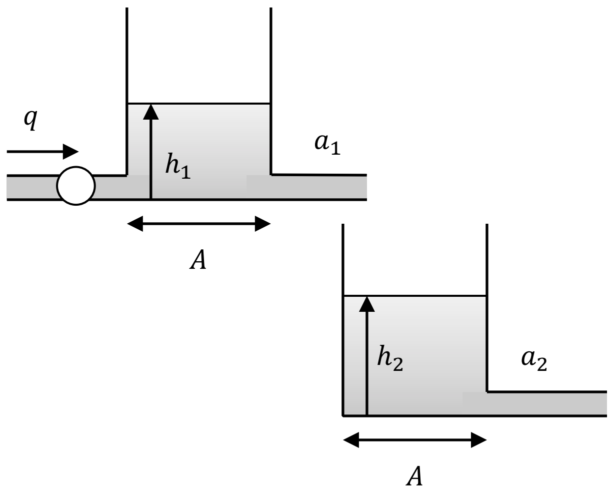

Figure 6: Two tanks system

4.2.1 Equations

The equations to describe the liquid level of this system are obtained after doing the mass balance of both tanks. The mass balance defines that ‘According to the *Principle of mass conservation,* the mass that enters a system must leave the system or accumulate within the system’ (Input = Output + Accumulation)[1].

In this system, the mass balance is described as *Accumulation* = *Input* – *Output*. For the first tank, the input part is the inlet flow *q*, the output is the depart liquid from the first tank according to its discharge coefficient, and the accumulation part is the volume of the liquid inside the first tank. On the other hand, the input part for the second tank is the depart liquid of the first tank. The other coefficients are the same as the first tank, in other words, the accumulation part is the liquid inside the second tank and the output part is the depart liquid of this second tank. The nonlinear equations of this system are:

$$A \cdot \frac{dh_1}{dt} = q - a_1 \cdot \sqrt{2 \cdot g \cdot h_1}$$
$$A \cdot \frac{dh_2}{dt} = a_1 \cdot \sqrt{2 \cdot g \cdot h_1} - a_2 \cdot \sqrt{2 \cdot g \cdot h_2} \qquad (13)$$

where q is the input variable, h_1 *and* h_2 are the state variables and there is not any output variable in this system.

The state equation of this system can be written as:

$$\begin{bmatrix} \dot{h_1} \\ \dot{h_2} \end{bmatrix} = \frac{1}{A} \cdot \begin{bmatrix} q - a_1 \cdot \sqrt{2 \cdot g \cdot h_1} \\ a_1 \cdot \sqrt{2 \cdot g \cdot h_1} - a_1 \cdot \sqrt{2 \cdot g \cdot h_1} \end{bmatrix} \qquad (14)$$

4.2.2 Parameters

In the following table there is a set of parameters which are used in this system.

Parameter / variable of two tanks system	Initial value
Cross section area of tanks, *A*	$0{,}03\ m^2$

[1]Definition of the mass balance: *Introducción a la Ingeniería Química*. [5]

Discharge coefficient of the first tank, a_1	$1{,}3104 \cdot 10^{-4}\ m^2$		
Discharge coefficient of the second tank, a_2	$1{,}5074 \cdot 10^{-4}\ m^2$		
Gravity constant, g	$9{,}81\ \frac{m}{s^2}$		
Liquid level of the first tank, h_1	$0{,}1\ m$		
Liquid level of the second tank, h_2	$0{,}2\ m$		
Inlet flow, q	$0{,}0003\ \frac{m^3}{s}$		
Initial value of h_1 and h_2, h_10, h_20	$0{,}1\ m$ and $0{,}2\ m$ respectively, can be modified by the user		
Equilibrium points	h_1*	$1.1{\times}h_1 0$	Can be modified by user
	h_2*	$\left(\frac{a_1}{a_2}\right)^2 \cdot (1.1{\times}h_1 0)$	
	q*	$a_1 \cdot \sqrt{2 \cdot g \cdot (1.1{\times}h_1 0)}$	
Mode	0	Open loop mode	
	1	Closed loop mode	
	2	Closed loop mode with observers	
State feedback gain, K_1 and K_2	Initially 0,01 and 0,02 respectively, can be modified by user		
Observer gain, L_1 and L_2	Initially 0, can be modified by user		
Auxiliary input, v	0		
Estimated state, $\widetilde{h_1}$ and $\widetilde{h_2}$	0,11 and 0,183 respectively		

Table 1: Parameters of two tanks system

4.2.3 Equilibriums points

As explained in chapter 3.1, equilibrium points are calculated by equalizing the state equations with zero. In this case, the equilibrium points are defined when the both tanks have the same value of input and output; that is to say, when the inlet flow of the first tank and its output flew are the same. As the output of the first tank is the inlet flew of the second tank, the output of the second tank should have the same value with the input of the first tank.

The equilibrium points of this system, namely h_1*, h_2* and q*, can be obtained:

$$\dot{h_1} = 0 \rightarrow -a_1 \cdot \sqrt{2 \cdot g \cdot h_1} + q = 0 \rightarrow \boldsymbol{a_1 \cdot \sqrt{2 \cdot g \cdot h_1} = q}$$

$$\dot{h_2} = 0 \rightarrow a_1 \cdot \sqrt{2 \cdot g \cdot h_1} - a_2 \cdot \sqrt{2 \cdot g \cdot h_2} = 0 \rightarrow \boldsymbol{a_1 \cdot \sqrt{2 \cdot g \cdot h_1} = a_2 \cdot \sqrt{2 \cdot g \cdot h_2}} \tag{15}$$

To calculate these equilibrium points, one of them has to be considered as variable, and the others two are calculated using the equations presented below:

Considering h_1 as variable:

$$h_2{}^*(h_1{}^*) = \left(\frac{a_1}{a_2}\right)^2 \cdot h_1{}^*$$

$$q^*(h_1{}^*) = a_1 \cdot \sqrt{2 \cdot g \cdot h_1{}^*} \tag{16}$$

Considering h_2 as variable:

$$h_1{}^*(h_2{}^*) = \left(\frac{a_2}{a_1}\right)^2 \cdot h_2{}^*$$

$$q^*(h_2{}^*) = a_2 \cdot \sqrt{2 \cdot g \cdot h_2{}^*} \tag{17}$$

Considering q as variable:

$$h_1{}^*(q^*) = \frac{q^{*2}}{2 \cdot g \cdot a_1{}^2}$$

$$h_2{}^*(q^*) = \frac{q^{*2}}{2 \cdot g \cdot a_2{}^2} \tag{18}$$

4.2.4 Linearized model of the two tanks system

To linearize the present system by Taylor expansion around the equilibrium points, is necessary define a set of new variables, these variables are defined as the displacement respect the equilibrium points and they are characterized by the subtraction of the state variable, input variable and output variables with their corresponded equilibrium points ($\boldsymbol{x}^*, u^*$ and y^*). In this case, $\boldsymbol{x}^* = ({h_1}^*, {h_2}^*)$ and $u^* = q^*$, there is no output variable y^*, so the new variable defined are:

$$\begin{aligned} \hbar_1 &= h_1 - {h_1}^* \\ \hbar_2 &= h_2 - {h_2}^* \\ \not{q} &= q - q^* \end{aligned} \tag{19}$$

Once defined these new variables, the next stage is to calculate the matrix ***A*** and ***B***, which are the *Jacobian Matrix* of the state equations ($\dot{h_1}, \dot{h_2}$):

$$\begin{aligned} f1 = \dot{h_1} &= \frac{-a_1 \cdot \sqrt{2 \cdot g \cdot h_1}}{A} + \frac{q}{A} \\ f2 = \dot{h_2} &= \frac{a_1 \cdot \sqrt{2 \cdot g \cdot h_1}}{A} - \frac{a_2 \cdot \sqrt{2 \cdot g \cdot h_2}}{A} \end{aligned} \tag{20}$$

The matrix ***A*** is:

$$\boldsymbol{A} = \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{x}} = \begin{bmatrix} \dfrac{\partial f1}{\partial h_1} & \dfrac{\partial f1}{\partial h_2} \\ \dfrac{\partial f2}{\partial h_1} & \dfrac{\partial f2}{\partial h_2} \end{bmatrix} = \begin{bmatrix} -\dfrac{1}{2} \cdot \dfrac{a_1 \cdot \sqrt{2} \cdot g}{A \cdot \sqrt{g \cdot h_1}} & 0 \\ -\dfrac{1}{2} \cdot \dfrac{a_1 \cdot \sqrt{2} \cdot g}{A \cdot \sqrt{g \cdot h_1}} & -\dfrac{1}{2} \cdot \dfrac{a_2 \cdot \sqrt{2} \cdot g}{A \cdot \sqrt{g \cdot h_2}} \end{bmatrix} \tag{21}$$

then replace the variables h_1 and h_2 to h_1* and h_2* respectively:

$$\boldsymbol{A} = \begin{bmatrix} -\dfrac{1}{2} \cdot \dfrac{a_1 \cdot \sqrt{2} \cdot g}{A \cdot \sqrt{g \cdot {h_1}^*}} & 0 \\ -\dfrac{1}{2} \cdot \dfrac{a_1 \cdot \sqrt{2} \cdot g}{A \cdot \sqrt{g \cdot {h_1}^*}} & -\dfrac{1}{2} \cdot \dfrac{a_2 \cdot \sqrt{2} \cdot g}{A \cdot \sqrt{g \cdot {h_2}^*}} \end{bmatrix} \tag{22}$$

the matrix ***B*** is:

$$\boldsymbol{B} = \frac{\partial \boldsymbol{f}}{\partial u} = \begin{bmatrix} \dfrac{\partial f1}{\partial q} \\ \dfrac{\partial f2}{\partial q} \end{bmatrix} = \begin{bmatrix} \dfrac{1}{A} \\ 0 \end{bmatrix} \tag{23}$$

The linearized form of two tanks system is:

$$\begin{bmatrix} \dot{\hslash}_1 \\ \dot{\hslash}_2 \end{bmatrix} = \begin{bmatrix} -\dfrac{1}{2} \cdot \dfrac{a_1 \cdot \sqrt{2} \cdot g}{A \cdot \sqrt{g \cdot {h_1}^*}} & 0 \\ -\dfrac{1}{2} \cdot \dfrac{a_1 \cdot \sqrt{2} \cdot g}{A \cdot \sqrt{g \cdot {h_1}^*}} & -\dfrac{1}{2} \cdot \dfrac{a_2 \cdot \sqrt{2} \cdot g}{A \cdot \sqrt{g \cdot {h_2}^*}} \end{bmatrix} \cdot \begin{bmatrix} \hslash_1 \\ \hslash_2 \end{bmatrix} + \begin{bmatrix} \dfrac{1}{A} \\ 0 \end{bmatrix} \cdot \not{q} \tag{24}$$

4.3 State feedback system

As described in chapter 3.3, the general form of the control law used to stabilize the equilibrium points of the present system is $u = -\boldsymbol{K} \cdot \boldsymbol{x}$, the assumption of all the state variables are available for feedback is necessary. Applying this control law to the displacement of the state variables (h_1 and h_2) respect their corresponding equilibrium points (h_1* and h_2*), the control law is:

$$q = q^* - K_1 \cdot (h_1 - {h_1}^*) - K_2 \cdot (h_2 - {h_2}^*) = q^* - K_1 \cdot \hslash_1 - K_2 \cdot \hslash_2 \tag{25}$$

With the control low, the linearized form of two tanks system can be written as:

$$\begin{bmatrix} \dot{\hslash}_1 \\ \dot{\hslash}_2 \end{bmatrix} = \left[\begin{bmatrix} -\dfrac{1}{2} \cdot \dfrac{a_1 \cdot \sqrt{2} \cdot g}{A \cdot \sqrt{g \cdot {h_1}^*}} & 0 \\ -\dfrac{1}{2} \cdot \dfrac{a_1 \cdot \sqrt{2} \cdot g}{A \cdot \sqrt{g \cdot {h_1}^*}} & -\dfrac{1}{2} \cdot \dfrac{a_2 \cdot \sqrt{2} \cdot g}{A \cdot \sqrt{g \cdot {h_2}^*}} \end{bmatrix} - \begin{bmatrix} \dfrac{1}{A} \\ 0 \end{bmatrix} [K_1 \quad K_2] \right] \cdot \begin{bmatrix} \hslash_1 \\ \hslash_2 \end{bmatrix} \tag{26}$$

4.4 State observer system

In case that some state variables are not available for feedback, the estimation of unavailable state variables, also called as observation, is need. As mentioned in the section 3.4, the mathematical model of observer is:

$$\dot{\widetilde{\boldsymbol{x}}} = \boldsymbol{A}\widetilde{\boldsymbol{x}} + \boldsymbol{B}u + \boldsymbol{L}(y - \boldsymbol{C}\widetilde{\boldsymbol{x}}) \tag{27}$$

Applying to the current system, where estimated state $\widetilde{\boldsymbol{x}} = \left(\widetilde{h_1}, \widetilde{h_2}\right)$ and the observer gain matrix $\boldsymbol{L} = \begin{pmatrix} L_1 \\ L_2 \end{pmatrix}$, consider h_2 as the output variable, the matrix $\boldsymbol{C}$ is calculated as:

$$\boldsymbol{C} = \frac{\partial y}{\partial \boldsymbol{x}} = \left[\frac{\partial h_2}{\partial h_1} \quad \frac{\partial h_2}{\partial h_2}\right] = [\,0 \quad 1\,] \tag{28}$$

the observer system became:

$$\begin{bmatrix} \dot{\widetilde{h_1}} \\ \dot{\widetilde{h_2}} \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} \cdot \frac{a_1 \cdot \sqrt{2} \cdot g}{A \cdot \sqrt{g \cdot {h_1}^*}} & 0 \\ -\frac{1}{2} \cdot \frac{a_1 \cdot \sqrt{2} \cdot g}{A \cdot \sqrt{g \cdot {h_1}^*}} & -\frac{1}{2} \cdot \frac{a_2 \cdot \sqrt{2} \cdot g}{A \cdot \sqrt{g \cdot {h_2}^*}} \end{bmatrix} \cdot \begin{bmatrix} \widetilde{h_1} \\ \widetilde{h_2} \end{bmatrix} + \begin{bmatrix} \frac{1}{A} \\ 0 \end{bmatrix} \cdot (q - q^*) + \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} \cdot \left((h_2 - {h_2}^*) - [0 \quad 1] \cdot \begin{bmatrix} \widetilde{h_1} \\ \widetilde{h_2} \end{bmatrix}\right) \tag{29}$$

The control law applied in state observer system is:

$$q = q^* - K_1 \cdot \widetilde{h_1} - K_2 \cdot \widetilde{h_2} \tag{30}$$

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4.5 Manual of the application

In this paragraph, the instruction of two tanks system's application will be explained. The structure of the applet for mobile devices of this system is the same as the web page version.

First of all, notice tat all the simulations created with Easy Javascript Simulation can be run in a Reader App[1] provided by the EjsS's designer. This reader applet is available for ISO and Android. The next figure shows the main page of the EjsS Reader App for ISO version.

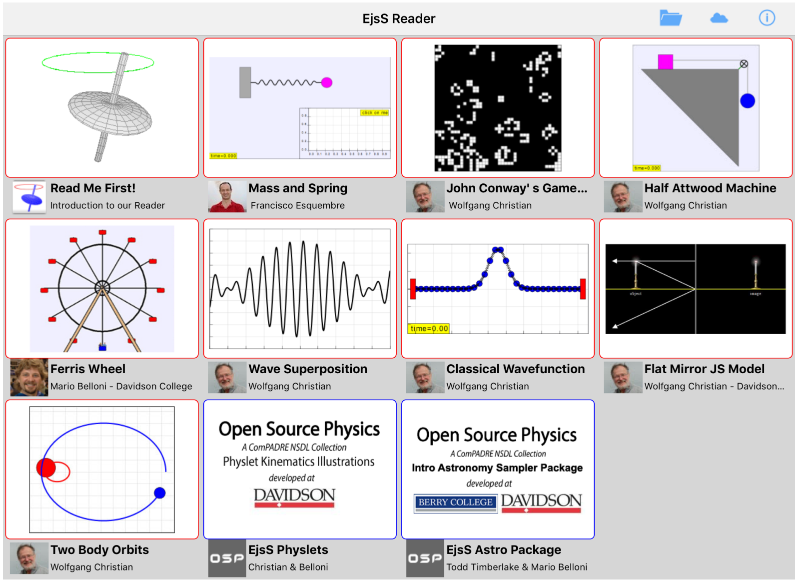

Figure 7: Main page of the EjsS Reader App for ISO version

[1] Ejs Wiki [1]

The following image presents the home page of two tanks system's application of a web site version.

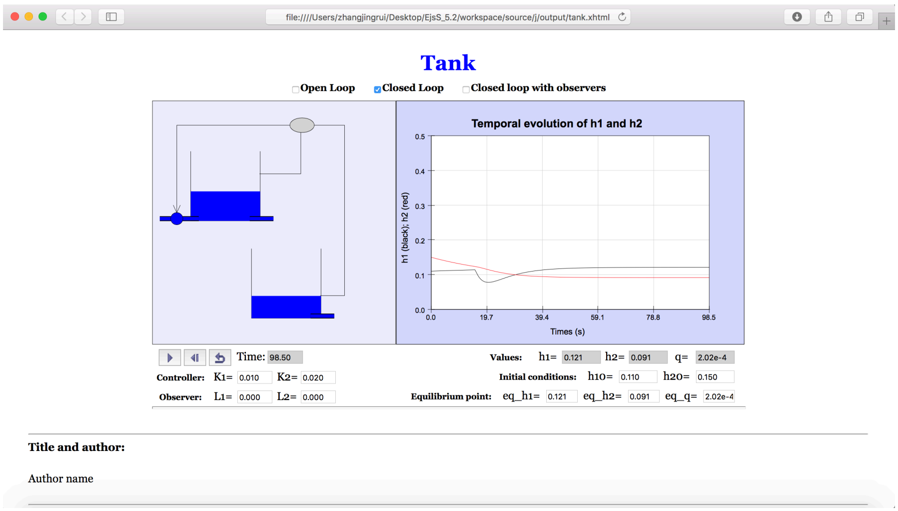

Figure 8: Two tanks system's home page for web site version

The figure below illustrates the home page for a smartphone with a screen-size of 5,5 inch, which corresponds to 139,7 millimetres. The picture on the left site corresponds to the general description of this system and the right one is the application's principal page. To change one page to another simply swipe right the smartphone's screen or click on the book icon on the top right corner of the screen and select the simulation or description page.

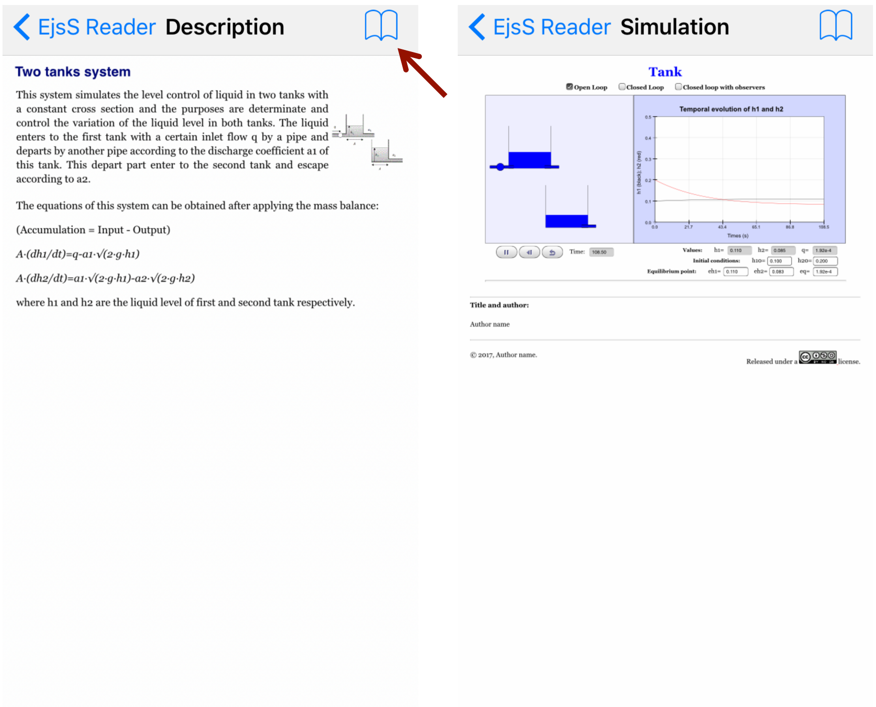

Figure 9: Home page of two tanks system executing by an ISO version's smartphone

On the top of the main page of this simulation, a multiple choices selector (Figure 10), which defines the different modes of this system, is introduced. There are three modes available, the open loop, the closed loop and the closed loop with observers. The corresponding representations of the modes specified are showed in figure 11. In this case, the mode selected is the closed loop.

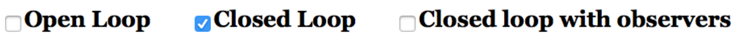

Figure 10: Mode selector

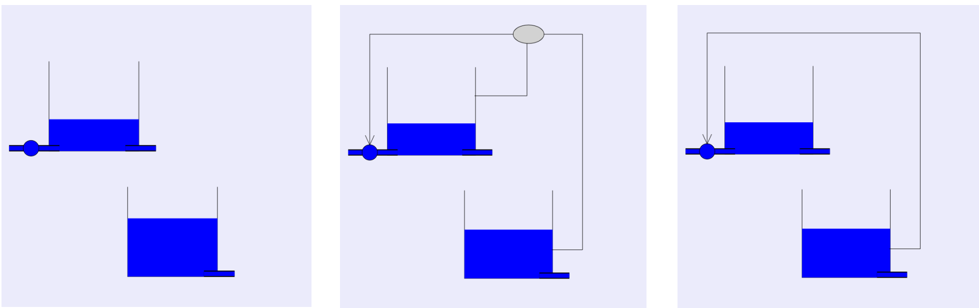

Figure 11: Representation of *Open Loop mode*, *Closed Loop mode* and *Closed Loop with observer* of two tanks system respectively

Next figure shows the temporal evolution of the liquid level of this system. The black line corresponds to the temporal evolution of the first tank's liquid level, h_1, and the red line represents the evolution of the liquid level of second tank, h_2. The axis x represents times in second.

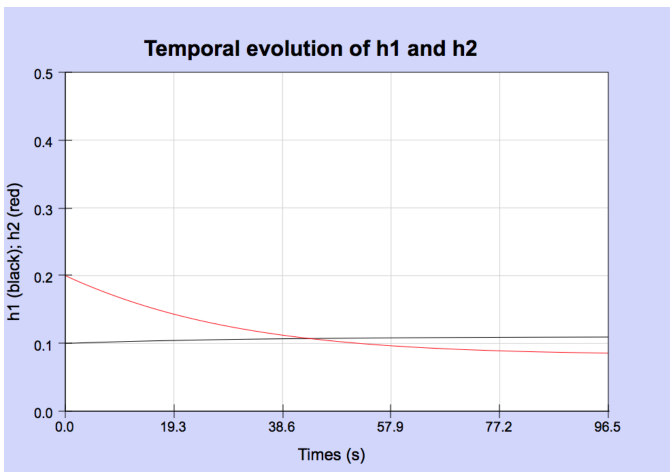

Figure 12: Time evolution of the liquid level in both tanks

The application also provides several edit fields and indicators fields. To change a value of any edit field, firstly enter the value and then click on the initialize button[1]. First of all, there is a control panel with a *PlayPause* button, an *Initialize* button and *Reset button* (Figure 13) on the left site, following by a time indicator (not editable); the time is shown in second and it is not editable.

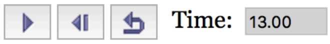

Figure 13: Control panel

On the right site, there is a value indicator (Figure 14) which is neither editable. The three boxes indicate the value of the liquid level of the first tank (h_1), the liquid level of the second tank (h_2) and the value of the input flow q.

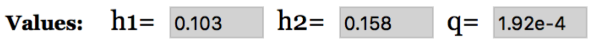

Figure 14: Value indicator of state variable and the input variable

In the initialization conditions edit fields (Figure 15), users can define the initial conditions h_10 and h_20 modifying the number inside the box; otherwise, the number of h_10 and h_20 will be the same as h_1 and h_2 respectively.

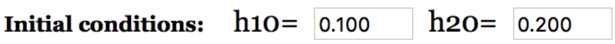

Figure 15:Initial condition edit field

The following panel on the right side is the editable panel of the equilibrium points (Figure 16). In this case, modifying one of the three values, the others two are automatically calculated and returned according to the equations defined on the chapter 3.2.3.

Figure 16: Edit panel for equilibrium points

On the left site, under the control panel, there are edit fields for the state feedback gain K_1 and K_2 (Figure 17) and below these there are others for observer gain L_1 and L_2 (Figure 18) which can be also modified by users.

[1] Notice that the initialization does no reset the time to *0 seconds*, and it has to be considered when we analyse the results of each application.

Controller: K1= 0.010 K2= 0.020

Figure 17: Edit field for state feedback gains K_1 and K_2

Observer: L1= 0.000 L2= 0.000

Figure 18: Edit field for observer gains L_1 and L_2

4.6 Example

4.6.1 Example of open loop

The first example illustrates a simulation of open loop mode of two tanks system, the initial conditions of liquid level of first tank $h_1 0$ and second tank $h_2 0$ and the equilibrium point h_1^* are showed in the table below.

Variables		Value
Initial condition	$h_1 0$	$0{,}11\ m$
	$h_2 0$	$0{,}15\ m$
Equilibrium point, h_1^*		$0{,}121\ m$

Table 2: Variables predetermined by user

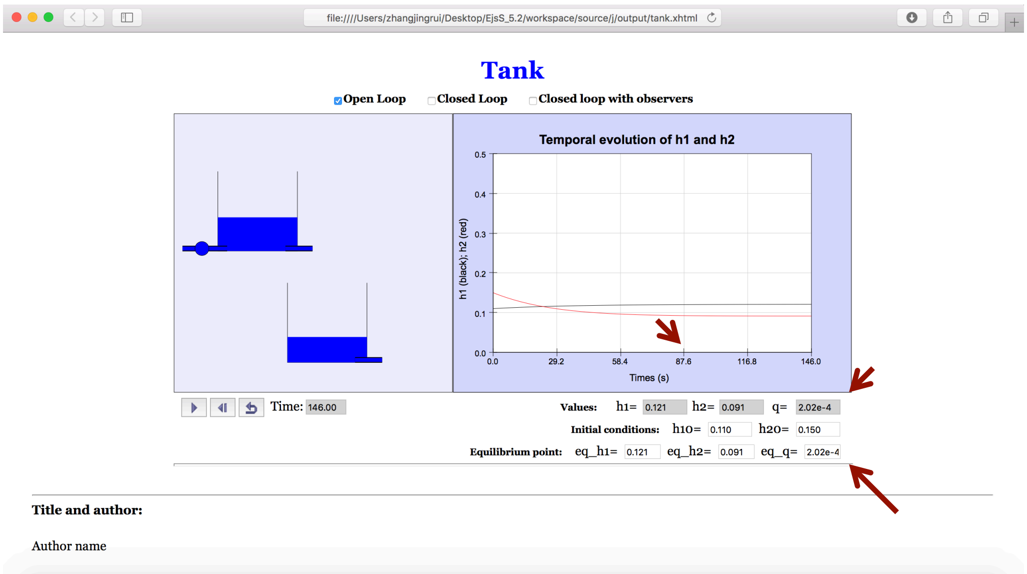

Figure 19: Example of open loop mode with initial conditions $h_1 0$=0,11 m, $h_2 0$=0,15 m and equilibrium point h_1^*=0,121 m

Once introduced the value of the equilibrium point given, the other values of the equilibrium points are automatically calculated and displayed in their corresponding fields according to the equations defined in chapter 4.2.3. In this case, as the variable chosen is ${h_1}^*$, $({h_1}^* = 0{,}121m)$, ${h_2}^*$ and q^* are calculated using equations (16) and the value obtained are $0{,}091\ m$ and $2{,}02\times 10^{-4}\ \frac{m^3}{s}$ respectively. The liquid levels of both tanks and the inlet flow *q* tend to their equilibrium points ($0{,}121\ m$, $0{,}091\ m$ and $2{,}02\times 10^{-4}\frac{m^3}{s}$ respectively) once arrive the steady state, showed in the section of value indicator and also in the graphic. The corresponding time for this system to reach the steady state of this system is approximately *87,6 sec*onds after initialize the simulation.

4.6.2 Example of closed loop

Considering the same parameter of initial conditions $h_1 0$, $h_2 0$ and equilibrium point $h_1{}^*$ with the open loop mode example in preview section, the liquid level of both tanks of closed loop mode presents the following temporal evolution (Figure 20). The state feedback gains, K_1 and K_2, in this case are $0{,}01$ and $0{,}02$ respectively.

In this case, the temporal evolution of liquid in first tank (h_1) presents a discharge after selecting closed loop mode. The reason is that there is no sufficient time for h_1 to achieve its equilibrium point. After this discharge, the value h_1 went back to tend to his equilibrium point again ($0.121\ m$). The controller used in this case is $q = 2{,}02\times 10^{-4} - 0,01 \cdot (h_1 - 0,121) - 0,02 \cdot (h_2 - 0,09)$[1]. The time needed to achieve the steady state is around 52,2 seconds, less than the case of open mode.

[1] Equation 25 defined in chapter 4.3.

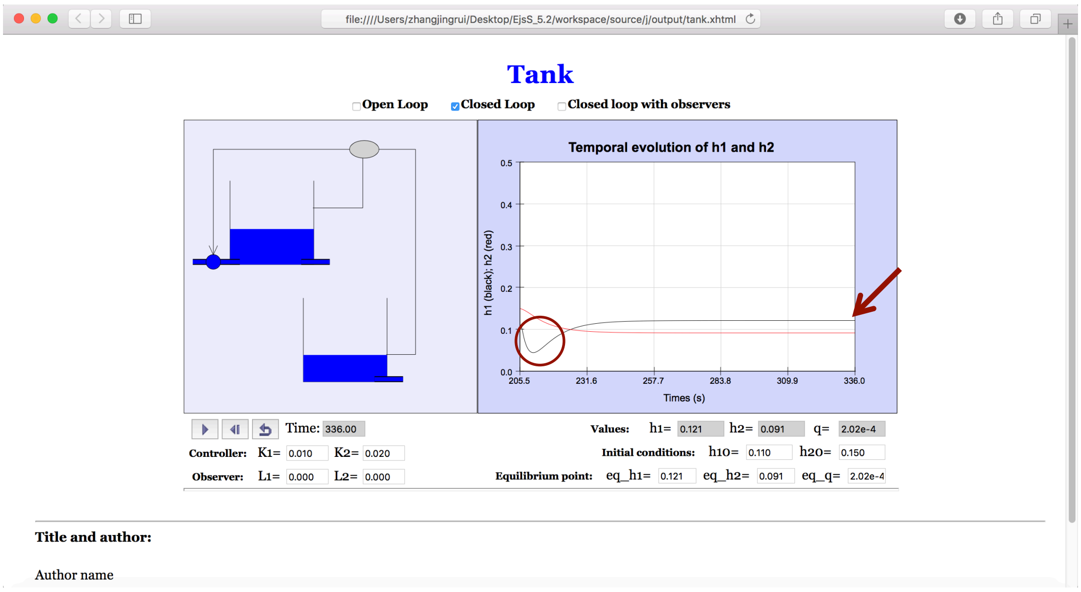

Figure 20: Example of closed loop with initial conditions $h_1 0=0{,}11$ m, $h_2 0=0{,}15$ m and equilibrium point $h_1^*=0{,}121$ m

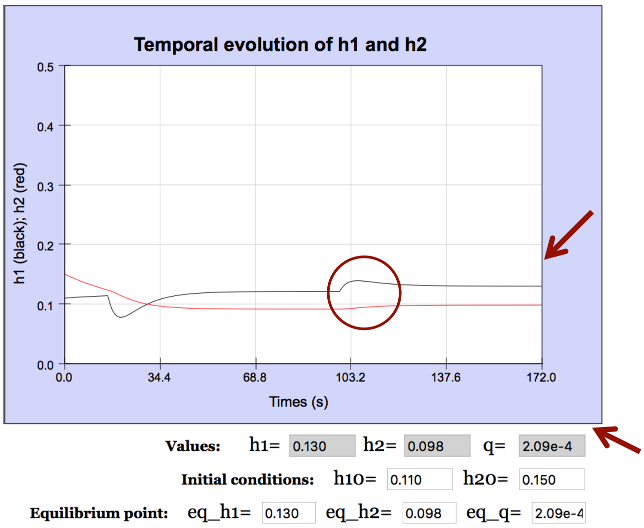

Figure 21: Temporal evolution of the closed loop example with a different equilibrium point ($h_1^*=0{,}130$ m)

After modifying the equilibrium points (considering that the given data of equilibrium point is $h_1{}^* = 0{,}13\ m$; $h_2{}^*$and q^* are automatically calculated and returned by the application, the corresponding value are $0{,}098\ m$ and $2{,}09{\times}10^{-4}\ \frac{m^3}{s}$ respectively), the liquid level of both tanks tends to their corresponding values of equilibrium points (figure 21). The liquid level of the first tank presents a charge immediately after alter its equilibrium point; the reason can be the more easily to achieve their equilibrium points.

4.6.3 Example of state observer analysis

The last example is an example of state observer analysis. The observer gains L_1 and L_2 are corresponding to 0,2 and 0,1 respectively and the others variables are the same as the examples presented in previous example.

Variables		Value
Initial condition	$h_1 0$	$0{,}11\ m$
	$h_2 0$	$0{,}15\ m$
Equilibrium point	$h_1{}^*$	$0{,}121\ m$
Estimated state	$\widetilde{h_1}$	$0{,}11\ m$
	$\widetilde{h_2}$	$0{,}083\ m$
State feedback gain	K_1	0,01
	K_2	0,02
Observer gain	L_1	0,02
	L_2	0,01

Table 3: Parameters used in state observer analysis of two tanks system

The controller and observer used in this mode are defined using the equations 29 and 30 applying the corresponding variables of this case.

The expressions of the temporal variation of estimated state of two tanks variables are calculated by:

$$\begin{bmatrix} \dot{\widetilde{h_1}} \\ \dot{\widetilde{h_2}} \end{bmatrix} = \begin{bmatrix} -\frac{1}{2}\cdot\frac{1{,}3104\times10^{-4}\cdot\sqrt{2}\cdot9{,}81}{0{,}03\cdot\sqrt{9{,}81\cdot0{,}121}} & 0 \\ -\frac{1}{2}\cdot\frac{1{,}3104\times10^{-4}\cdot\sqrt{2}\cdot9{,}81}{0{,}03\cdot\sqrt{9{,}81\cdot0{,}121}} & -\frac{1}{2}\cdot\frac{1{,}5074\times10^{-4}\cdot\sqrt{2}\cdot9{,}81}{0{,}03\cdot\sqrt{9{,}81\cdot0{,}091}} \end{bmatrix}\cdot\begin{bmatrix} 0{,}11 \\ 0{,}083 \end{bmatrix} +$$

$$\begin{bmatrix} \frac{1}{0{,}03} \\ 0 \end{bmatrix}\cdot\left(q-2{,}02\times10^{-4}\right)+\begin{bmatrix} 0{,}02 \\ 0{,}01 \end{bmatrix}\cdot\big((h_2-0{,}091)-0{,}083\big) \qquad (31)$$

$$\dot{\widetilde{h_1}} = -0{,}001 + 33{,}33\times(q - 2{,}02\times10^{-4}) + 0{,}02\times\big((h_2 - 0{,}091) - 0{,}083\big)$$

$$\dot{\widetilde{h_2}} = -0{,}00407 + 0\times(q - 2{,}02\times10^{-4}) + 0{,}01\times\big((h_2 - 0{,}091) - 0{,}083\big)$$

And the control law applied in this case is:

$$q = 2{,}02\times10^{-4} - 0{,}01\cdot 0{,}11 - 0{,}02\cdot 0{,}083 \quad (32)$$

The following figure illustrates the temporal variation of h_1 and h_2.

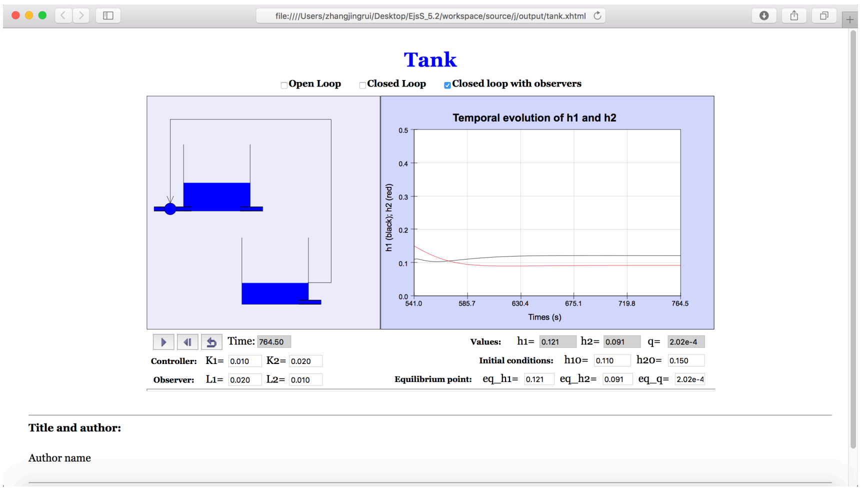

Figure 22: Example for state observer of two tanks system with L_1=0,02 and L_2=0,01

As showed, the system requires almost 89,4 seconds to reach the steady state.

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5. Pendulum

5.1 Description

In this chapter the study of a Pendulum system is realized. The objective is modelling the dynamic performance of a pendulum. The current study involves three modes, the open-loop mode, the closed-loop mode (state feedback system) and the closed loop mode with observers (state observer system).

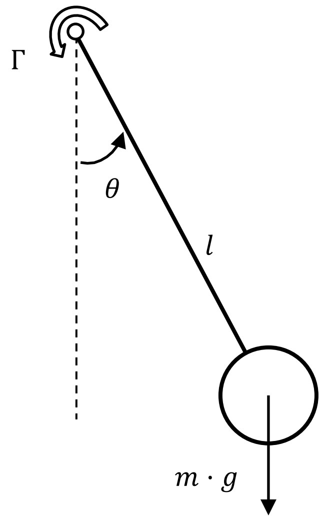

Figure 23: Pendulum

The image above illustrates the behaviour of a pendulum system. This system is composed by a ball with mass *m* situated in the extreme of a bar of length *l* and an insignificant mass. In addition, the moment Γ applied at the origin, the moment of inertia of the pendulum respects its rotated point, $m \cdot l^2$, and the friction coefficient at the hinge *b* are given.

5.2 Study of no lineal equations

5.2.1 Equations

The equation to describe the dynamic behaviour of the pendulum is obtained after doing the second equilibrium condition respects the origin. The second equilibrium condition announces that the addition of moments respect a point is null.

The equation used to describe the pendulum's dynamic behaviour is:

$$m \cdot l^2 \cdot \frac{d^2\,\theta}{dt^2} + b \cdot \frac{d\,\theta}{dt} + mgl \cdot \sin\,(\theta) = \Gamma \tag{33}$$

which is a second order Ordinary Differential Equation (ODE), this second order ODE can be transformed in to a first order ODEs by changing the name of the variable θ to x_1 and adding a new variable x_2.

$$\begin{aligned} x_1 &= \theta \\ x_2 &= \dot{x_1} = \dot{\theta} \end{aligned} \tag{34}$$

Then the first order ODEs can be written as:

$$\begin{aligned} \dot{x_1} &= x_2 \\ \dot{x_2} &= \frac{1}{m \cdot l^2} \cdot \Gamma - \frac{b}{m \cdot l^2} \cdot x_2 - \frac{g}{l} \cdot \sin(x_1) \end{aligned} \tag{35}$$

notice that the first order ODEs have the same form as a state space system. Where Γ is the input variable and x_1 and x_2 are state variables. The output equation in this case is:

$$y = x_1 \tag{36}$$

The state equations of this system can be written as:

$$\begin{aligned} \begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} &= \begin{bmatrix} x_2 \\ -\frac{g}{l} \cdot \sin(x_1) - \frac{b}{m \cdot l^2} \cdot x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m \cdot l^2} \cdot \Gamma \end{bmatrix} \\ y &= x_1 \end{aligned} \tag{37}$$

5.2.2 Parameters

In the following table there is a set of parameters which are used in this system.

Parameter / variable of two tanks system	Value
Ball mass, m	$0{,}1\ kg$

Pendulum's length, *l*	$0{,}3\ m$
Friction coefficient at the hinge, *b*	$0{,}1\ \frac{kg}{s}$
Gravity constant, *g*	$9{,}81\ \frac{m}{s^2}$
Moment, $\boldsymbol{\Gamma}$	$0{,}1\ N\cdot m$
Rotated angel, $\boldsymbol{\theta = x_1}$	$1\ rad$
Angular velocity, $\boldsymbol{\dot{\theta} = x_2}$	$0\ \frac{rad}{s}$
Initial value of state variables, x_10, x_20	1 and 0 respectively, can be modified by the user
Equilibrium points	x_1^*: $0\ rad$ x_2^*: $0\ \frac{rad}{s}$ Γ^*: $0\ N\cdot m$ Can be modified by user
Mode	0: Open loop mode 1: Closed loop mode 2: Closed loop mode with observers
State feedback gain, K_1 and K_2	Initially 0,01 and 0,02 respectively, can be modified by user
Observer gain, L_1 and L_2	Initially 0, can be modified by user
Auxiliary input, *v*	0
Estimated state, $\widetilde{x_1}$ and $\widetilde{x_2}$	$1{,}1\ rad$ and $0{,}1\ \frac{rad}{s}$ respectively

Table 4: Parameters of pendulum system

5.2.3 Equilibriums points

Equilibrium points are calculated by equalizing the state equations with zero. This system presents three equilibrium points, the first one correspond physically to the vertical position of the pendulum, when the angular velocity $\dot{x_2} = 0\frac{rad}{s}$ and the others equilibrium points are obtained using the expression $\frac{1}{m \cdot l \cdot g} \cdot \Gamma = \sin x_1$. The equilibrium points of this system are $x_1{}^*, x_2{}^*$ and Γ^*.

Equilibrium points of this system are obtained by:

$$\dot{x_1} = 0 \rightarrow \boldsymbol{x_2{}^* = 0}$$

$$\dot{x_2} = 0 \rightarrow \frac{1}{m \cdot l^2} \cdot \Gamma - \frac{b}{m \cdot l^2} \cdot x_2 - \frac{g}{l} \cdot \sin x_1 = 0 \rightarrow \frac{1}{m \cdot l^2} \cdot \Gamma = \frac{g}{l} \cdot \sin x_1 \qquad (38)$$

$$\boldsymbol{sin\, x_1{}^* = \frac{\Gamma^*}{m \cdot l \cdot g}}$$

5.2.4 Linearized model of the two tanks system

To linearize the present system applying Taylor expansion, is necessary define a set of new variables, these variables are defined as the displacement respect the equilibrium points and they are characterized by the subtraction of the state variable, input variable and output variables with their corresponded equilibrium points (x^*, u^* and y^*). In this case, $\boldsymbol{x}^* = (x_1{}^*, x_2{}^*)$, $u^* = \Gamma^*$ and the output variable $y^* = x_1{}^*$, so the new variable defined are:

$$\chi_1 = x_1 - x_1{}^*$$

$$\chi_2 = x_2 - x_2{}^* \qquad (39)$$

$$\mathcal{T} = \Gamma - \Gamma^*$$

Once defined these new variables, the next stage is to calculate the state matrix ***A***, the input matrix ***B***, which are the *Jacobian Matrix* of the state equations $(\dot{x_1}, \dot{x_2})$, and then replace the variables x_1 and x_2 to x_1* and x_2* respectively. In this case the matrixes ***C*** (output matrix) and ***D*** (direct transmission matrix) are also required in the linearization process because the output variable is not null.

$$f1 = \dot{x_1} = x_2$$

$$f2 = \dot{x_2} = -\frac{g}{l} \cdot \sin(x_1) - \frac{b}{m \cdot l^2} \cdot x_2 + \frac{1}{m \cdot l^2} \cdot \Gamma \tag{40}$$

The matrixes ***A*** and ***B*** are:

$$\boldsymbol{A} = \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{x}} = \begin{bmatrix} \frac{\partial f1}{\partial x_1} & \frac{\partial f1}{\partial x_2} \\ \frac{\partial f2}{\partial x_1} & \frac{\partial f2}{\partial x_2} \end{bmatrix}_{x_1 = x_1{}^*} = \begin{bmatrix} 0 & 1 \\ -\frac{g}{l} \cdot \cos(x_1{}^*) & -\frac{b}{m \cdot l^2} \end{bmatrix} \tag{41}$$

$$\boldsymbol{B} = \frac{\partial \boldsymbol{f}}{\partial u} = \begin{bmatrix} \frac{\partial f1}{\partial \Gamma} \\ \frac{\partial f2}{\partial \Gamma} \end{bmatrix} = \begin{bmatrix} 0 \\ \frac{1}{m \cdot l^2} \end{bmatrix} \tag{42}$$

The matrix ***C*** and ***D*** are calculated by:

$$\boldsymbol{C} = \frac{\partial h}{\partial \boldsymbol{x}} = \begin{bmatrix} \frac{\partial h}{\partial x_1} & \frac{\partial h}{\partial x_2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$\boldsymbol{D} = \frac{\partial h}{\partial \Gamma} = \begin{bmatrix} \frac{\partial h}{\partial \Gamma} \end{bmatrix} = 0 \tag{43}$$

The linear system is:

$$\begin{bmatrix} \dot{\mathcal{X}_1} \\ \dot{\mathcal{X}_2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{g}{l} \cdot \cos(x_1{}^*) & -\frac{b}{m \cdot l^2} \end{bmatrix} \cdot \begin{bmatrix} \mathcal{X}_1 \\ \mathcal{X}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m \cdot l^2} \end{bmatrix} \cdot \mathcal{T}$$

$$\mathcal{X}_1 = \begin{bmatrix} 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} \mathcal{X}_1 \\ \mathcal{X}_2 \end{bmatrix} + 0 \tag{44}$$

5.3 State feedback system

The general form of the control law used to stabilize the equilibrium points is $u = -\boldsymbol{K} \cdot \boldsymbol{x}$, mentioned in chapter 3.3. Applying this control law to the displacement of the state variables (x_1 and x_2) respect their corresponding equilibrium points (x_1* and x_2*), the control law is:

$$\Gamma = \Gamma^* - K_1 \cdot (x_1 - x_1{}^*) - K_2 \cdot (x_2 - x_2{}^*) \tag{45}$$

5.4 State observer system

In case that some state variables are not available for feedback, the estimation of unavailable state variables, also called as observation, is need. The mathematical model of observer used is mentioned in section 3.4, and applying to the current system, where the estimated state $\widetilde{\boldsymbol{x}} = (\widetilde{x_1}, \widetilde{x_2})$ and the observer gain matrix $\boldsymbol{L} = \begin{pmatrix} L_1 \\ L_2 \end{pmatrix}$, the observer system became:

$$\begin{bmatrix} \dot{\widetilde{x_1}} \\ \dot{\widetilde{x_2}} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{g}{l} \cdot \cos(x_1{}^*) & -\frac{b}{m \cdot l^2} \end{bmatrix} \cdot \begin{bmatrix} \widetilde{x_1} \\ \widetilde{x_2} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m \cdot l^2} \end{bmatrix} (\Gamma - \Gamma^*) + \begin{bmatrix} L_1 \\ L_2 \end{bmatrix} \cdot \left((x_1 - x_1{}^*) - [\,1 \quad 0\,] \cdot \begin{bmatrix} \widetilde{x_1} \\ \widetilde{x_2} \end{bmatrix} \right) \tag{46}$$

The control law applied in this state observer system is:

$$\Gamma = \Gamma^* - K_1 \cdot \widetilde{x_1} - K_2 \cdot \widetilde{x_2} \tag{47}$$

5.5 Manual of the application

In this paragraph, the instruction of application of pendulum will be explained.

First of all, the following images present the home page of pendulum's application. The figure 24 corresponds the homepage of WWW and the figure 25 for mobile devices, in this case, it is a smartphone with screen size of 5,5 inch. As can be seen, the simulation page of these two versions have the same structure.

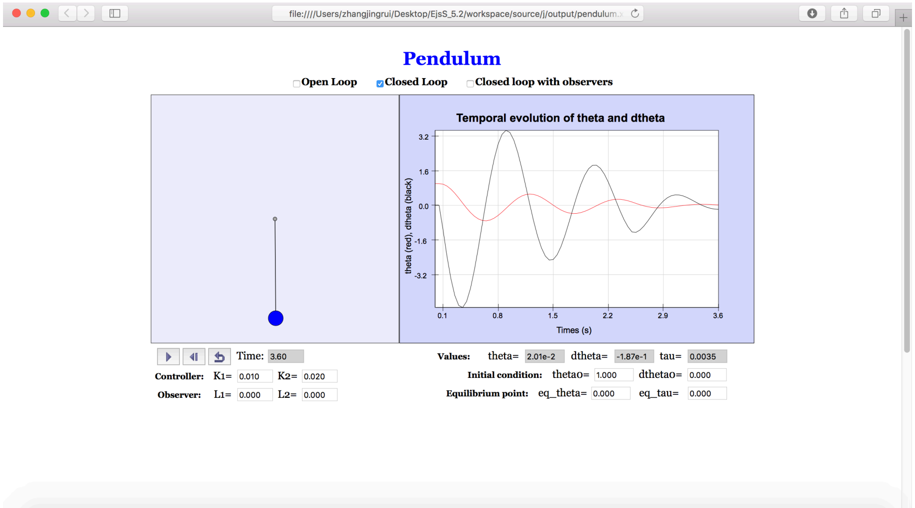

Figure 24: Pendulum's homepage for web site version

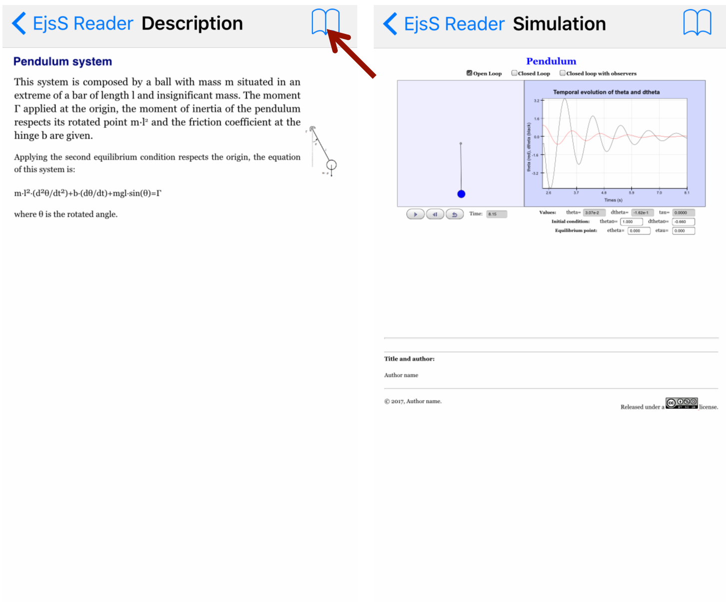

Figure 25: Homepage of pendulum executing using an ISO version's smartphone

In figure 25, the picture on the left site corresponds to a general description of this system and the right one is the application's principal page. To change one page from another simply swipe right the smartphone's screen or click on the book icon on the top right corner of the screen to select the description or simulation page.

On the top panel of the main page of pendulum, a multiple choices selector (Figure 26), which defines the different modes of this system, is introduced. There are three modes available, the open loop, the closed loop and the closed loop with observers. In this case, the mode selected is the closed loop and these three modes of pendulum have the same representation (Figure 27).

Figure 26: Mode selector

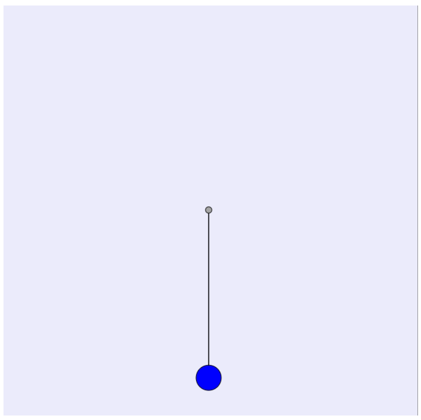

Figure 27: Representation of *Open Loop mode*, *Closed Loop mode* and *Closed Loop with observer* of pendulum.

Next figure shows the temporal evolution of the state variables of this system, namely the rotated angel x_1 and its angular velocity x_2. The red line represents the temporal evolution of the rotated angel, and the black line corresponds the evolution of the angular velocity. The axis x represents times in second.

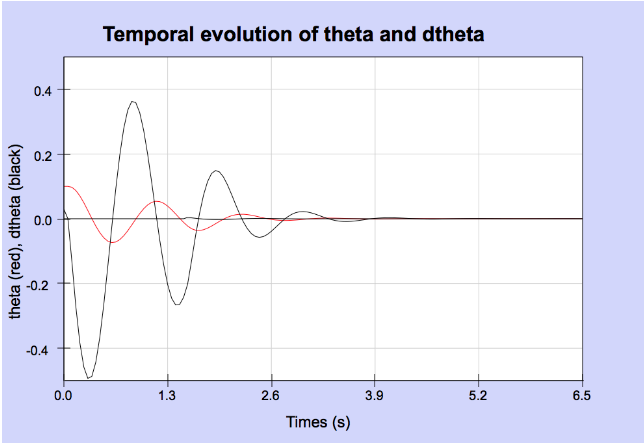

Figure 28: Time evolution of the state variables of pendulum system

The application also provides several edit fields and indicators fields. Firstly, there is a control panel with a *PlayPause* button, an *Initialize* button and *Reset button* (Figure 29) on the left site, following by a time indicator (not editable). The time is shown in second.

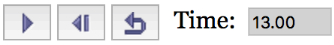

Figure 29: Control panel

Below the graphic of the temporal evolution of state variables of this system, there is a value indicator (Figure 30) which is not editable. The three boxes indicate the value of the rotated angel $x_1\,(theta)$, its angular velocity $x_2(dtheta)$ and the value of the moment Γ (tau).

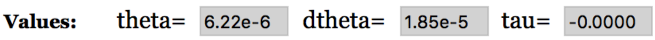

Figure 30: Value indicator of the state variables and input variable

In the initialization conditions edit field (Figure 31), users can define the initial conditions $x_1 0\,(theta0)$ and $x_2 0\,(dtheta0)$ modifying the number inside the box; otherwise, the number of $x_1 0$ and $x_2 0$ will be the same as x_1 and x_2 respectively.

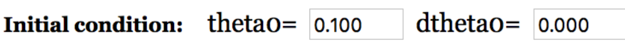

Figure 31: Initial condition's edit field

The following panel, below the initial condition, is the editable panel of the equilibrium points (Figure 32) $x_1{}^*(eq\_theta)$ and $\Gamma^*(eq\_tau)$. In this case, as the equilibrium point $x_2{}^*$ is always equal to zero, this equilibrium point is no represented. Modifying one of the values, the other is automatically calculated and returned to its corresponding field according to the equation $sin\, x_1{}^* = \frac{\Gamma^*}{m \cdot l \cdot g}$ defined on the chapter 5.2.3. The maximum value of equilibrium point of the moment Γ^* is $0.23\ N \cdot m$, which means the corresponding equilibrium point of rotated angle $x_1{}^*$, $0.9\ \mathrm{rad}$ approximately. Once overcome this limitation, this open loop mode of pendulum system became unstable; the value of rotated angle tends to increase in exponential form and its angular velocity tends to increase linearly until reach a double-digit value.

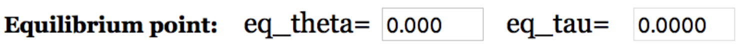

Figure 32: Edit panel for equilibrium points

On the left site, there are edit fields for the state feedback gain K_1 and K_2 and below these edit fields there are others for observer gain L_1 and L_2, also editable.

Figure 33: Edit field for state feedback gains

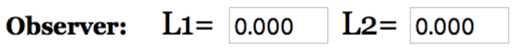

Figure 34: Edit field for observer gains

5.6 Example

5.6.1 Example of open loop

The first example simulates the open loop mode of this system; the initial conditions of the state variables, which are the rotated angle and the angular velocity, and the equilibrium point of the rotated angle are showed in the table below.

Variables		Value
Initial condition	$x_1 0(theta0)$	$1\ rad$
	$x_2 0(dtheta0\ )$	$0\ \frac{rad}{s}$
Equilibrium point, $x_1{}^*(eq\_theta)$		$0\ rad$

Table 5: Variables of pendulum predetermined by user

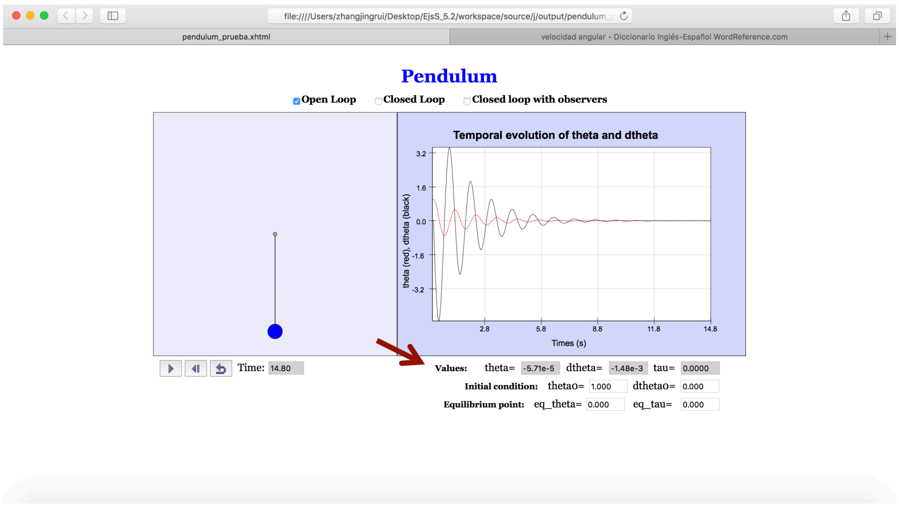

Figure 35: Example of open loop mode with initial conditions $x_1 0 = 1\ \mathrm{rad}$ and $x_2 0 = 0\ \frac{\mathrm{rad}}{\mathrm{s}}$

As seen in figure above, the value of state variables (x_1 and x_2) and the input variable Γ tend to their corresponding equilibrium points (*0 rad*, *0 rad/s* and *0 N·m* respectively) once arrived the steady state.

After modifying the value of the equilibrium point of $x_1{}^*$, the value of Γ^* is automatically calculated and returned in its corresponding field according to the equation $sin\, x_1{}^* = \frac{\Gamma^*}{m \cdot l \cdot g}$ demonstrated in the chapter 5.2.3.

The example in follow image illustrates the case after changing an equilibrium point, $x_1{}^* = 0{,}5\, rad$ and the value of corresponding Γ^* calculated by the application is $0{,}141\ N \cdot m$. The value of state variables (x_1 and x_2) and the input variable (Γ) tend to their corresponding equilibrium points in steady state, $0{,}5\, rad$, $0\ \frac{rad}{s}$[1] and $0{,}141\ N \cdot m$ respectively. As can be seen in figure 36, the time needed to reach the steady state is *17,5 seconds*.

[1] As explained in chapter 5.2.3, the equilibrium point of x_2 is always 0.

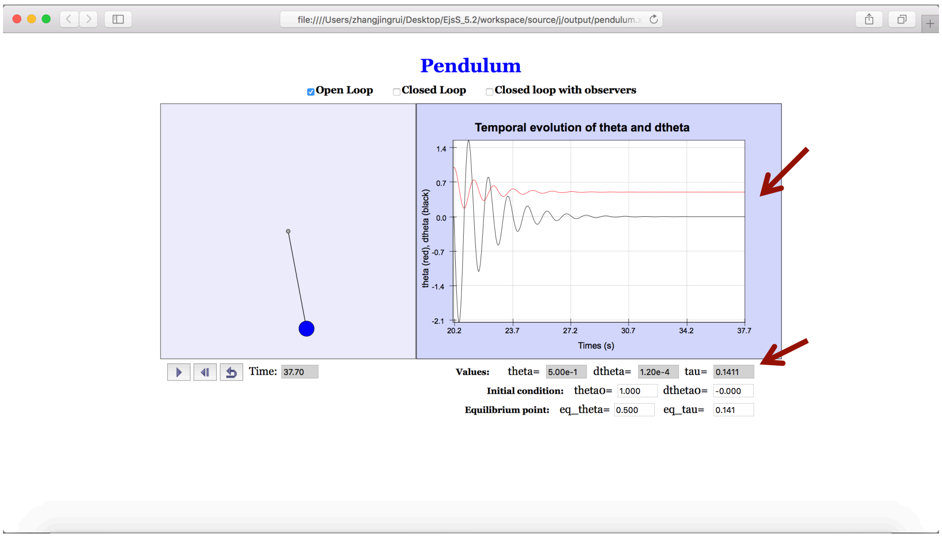

Figure 36: Open loop mode after changing the value of the equilibrium point $x_1{}^* = 0,5\ \mathrm{rad}$

5.6.2 Example of closed loop of pendulum

Considering the same parameter of initial conditions of state variables ($x_1 0$ and $x_2 0$) and equilibrium point of the rotated angle $x_1{}^*$ with the open loop mode example in preview section (1 rad, 0 rad/s and 0,5 rad respectively), the rotated angel and angular velocity of closed loop mode presents the following temporal evolution. The state feedback gains, K_1 and K_2, in this case are $0{,}01$ and $0{,}02$ respectively.

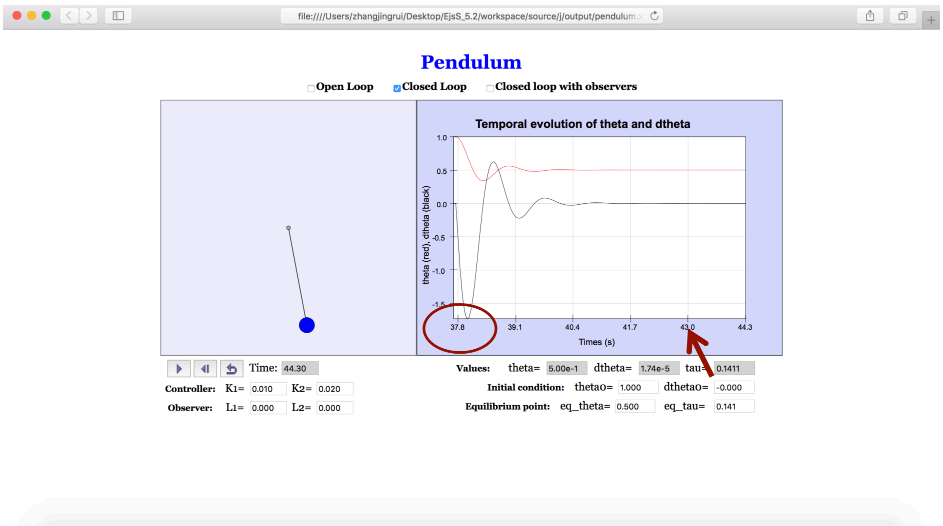

Figure 37: Example of closed loop with initial conditions $x_1 0 = 1\ \mathrm{rad}, x_2 0 = 0\,\frac{\mathrm{rad}}{\mathrm{s}}$ and equilibrium point $x_1{}^* = 0,5\ \mathrm{rad}$

As presented in the graphic of the temporal variation of x_1 and x_2 above, the present system takes less time to reach the steady state compared to the open loop mode. This closed loop with controller[1] requires *5,2* seconds approximately to reach the steady state (notice that the axis x start in *37,7 s*) whereas the open loop mode needs almost *17,5* seconds (Figure 35).

5.6.3 Example of state observer analysis of pendulum

The last example is an example of state observer analysis. The observer gains L_1 and L_2 are corresponding to $0{,}2$ and $0{,}1$ respectively and the others variables are the same as the examples presented in previous examples. The parameters used are represented in following table.

Variables		Value
Initial condition	$x_1 0(theta0)$	$1\,rad$
	$x_2 0(dtheta0\,)$	$0\,\frac{rad}{s}$
Equilibrium point	$x_1{}^*(eq\_theta)$	$0{,}5\,rad$
Estimated state	$\widetilde{x_1}$	$1{,}1\,rad$
	$\widetilde{x_2}$	$0{,}1\,\frac{rad}{s}$
State feedback gain	K_1	$0{,}01$
	K_2	$0{,}02$
Observer gain	L_1	$0{,}2$
	L_2	$0{,}1$

Table 6: Variables used in state observer analysis of Pendulum

The following figure illustrates the temporal variation of x_1 and x_2 of this system. The controller and observer used in this mode are defined using the equations 46 and 47 applying the corresponding variables of this example.

[1] $\Gamma = 0{,}141 - 0{,}01 \cdot (x_1 - 0{,}5) - 0{,}02 \cdot (x_2 - 0)$

The temporal variation of estimated states of pendulum are obtained by the following equations:

$$\begin{bmatrix} \dot{\tilde{x_1}} \\ \dot{\tilde{x_2}} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{9{,}81}{0{,}3} \cdot \cos(0{,}5) & -\frac{0{,}1}{0{,}1 \cdot 0{,}3^2} \end{bmatrix} \cdot \begin{bmatrix} 1{,}1 \\ 0{,}1 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{0{,}1 \cdot 0{,}3^2} \end{bmatrix} (\Gamma - 0{,}141) + \begin{bmatrix} 0{,}2 \\ 0{,}1 \end{bmatrix} \cdot \left((x_1 - 0{,}5) - \begin{bmatrix} 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1{,}1 \\ 0{,}1 \end{bmatrix} \right) \tag{48}$$

$$\dot{\tilde{x_1}} = 0{,}1 + 0 + 0{,}2 \times ((x_1 - 0{,}5) - 1{,}1)$$

$$\dot{\tilde{x_2}} = -32{,}6778 + 111{,}11 \times (\Gamma - 0{,}141) + 0{,}1 \times ((x_1 - 0{,}5) - 1{,}1)$$

And the control law applied in this example is:

$$\Gamma = 0{,}141 - 0{,}01 \cdot 1{,}1 - 0{,}02 \cdot 0{,}1 \tag{49}$$

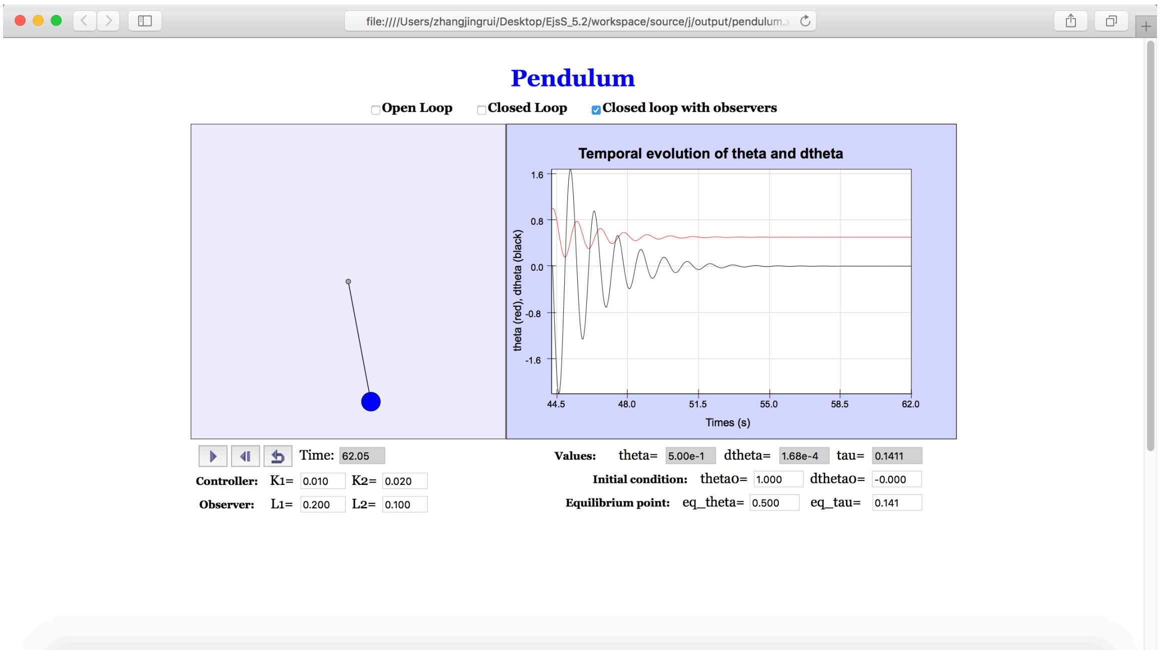

Figure 38: Example for state observer of two tanks system with L_1=0,2 and L_2=0,1

In this case, in order to achieve the steady state, the time needed is *14,1 seconds* (from *44,4 s* to *62 s* approximately).

6. Project budget

This chapter elaborates the economical study of this project. The project budget calculation is realized based on two types of costs, namely material costs and personal costs.

6.1 Personal costs

To calculate the human resources costs of this Bachelor's Thesis is necessary to do a schedule of the realization of this project during the last four months. First of all, the main tasks are:

- Learning how to use the Easy Javascript Simulation.
- Review of the Javascript Code.
- Design of the virtual laboratories; total time involved in creating both simulations, including the versions for web site and mobile devices of two tanks system and Pendulum. It is an approximated time in designing the prototypes of these simulations and times dedicated in improvements to reach the final version.
- Meeting with tutor.
- Do the writing paper.

The table below shows the approximate amount of time dedicated in every task during the realization of this project and the personal budget of this project.

Tasks	Hours[1]	€ / Hour	€
Learning how to use EjsS	*30*	*20*	*600*
Review of Javasript code	*10*	*15*	*150*
Design of the virtual laboratories	*160*	*20*	*3200*
Meeting with tutor	*20·2*	*20*	*800*
Do the writing paper	*60*	*20*	*1200*
Subtotal amount of personal budget			***5950 €***

Table 7: Project schedule and personal costs

[1] An approximate times in realization of each task.

6.2 Material costs

The direct and indirect material costs are included during the calculation of material costs. The direct material costs are based on the amortization of the equipment and materials used during the realization of this project, because of the program of Easy Javascript Simulation and the reader applet for mobile devices used during the realization of this project are free, they are not included in the calculation of the direct costs. The indirect costs are electricity and Internet connection costs.

The amortization of the equipment and materials is calculated by the following equation:

$$amortization = \frac{total\ cost}{service\ time} \cdot time\ used\ for\ the\ project \tag{50}$$

The following tables illustrate the material's economic study:

	Hours	€/hour	€
Personal computer	-	-	*250 €*[1]
Office supplies	-	-	*100 €*
Electricity	*290 hours*[2]	$40\frac{€}{month}$	*16,11 €*
Internet connection	*290 hours*	$47\frac{€}{month}$	*18,93 €*
	Subtotal		***385,041€***

Table 8: Material and equipment costs

[1] The cost of personal computer is calculated according to the equation of amortization:

$$\frac{1500€}{2\ years} \cdot \frac{4\ months}{12\ \frac{months}{year}} = 250€$$

[2] Total time dedicated in the realization of this Project according to the project schedule of chapter 6.1.

Finally, the total budget of this Bachelor's thesis is presented in following table:

Concept	**€**
Personal budget	*5950 €*
Material/ equipment costs	*385,041 €*
Total	***6335,041 €***

Table 9: Total budget in realization of the present project

7. Environmental impact

The present chapter consists in the realization of the study about the environmental impact of current project.

Nevertheless, as this project is about creating virtual laboratories using the software tool Easy Javascript Simulation, the methodology used during its realization and all the tasks realized to complete the current project are via Internet, the only way to damage the environment is the electricity. There is not such a large amount environment damage caused by accomplish the present project.

Conclusion

After finishing this current project, some of the conclusions:

- The accomplishment of the creation of two virtual laboratories using EjsS for web browser and mobile devices are finished. These two virtual laboratories are 'Two tanks system' and 'Pendulum'.
- Assuming the fact that the programing language provided by EjsS is Javascript, make the possibility of the assessment by not only the web browser, but also the mobile devices to simulations created using this software tool, which is a very interesting issue nowadays in education.
- Several studies of a nonlinear system can be realized with these virtual laboratories created in this project. Specifically, the study of the open loop of these two simulations, the analysis of the state feedback and also for state observer system.
- The virtual laboratories designed in this project can be used as a guideline to create and structure other simulations changing necessaries variables, equations and others elements.

Even though have finalized this present project accomplished all the objectives; there are still some future works which can be realized to enhance the applications. For example, the improvement of some Jacascript code, such as settle limitations for each variables using Javascript code, and adding more study for a nonlinear system. Moreover, the other kind of future work can be the creation of simulations for mobile devices using EjsS such as magnetic levitator.

Acknowledgements

The achievement of this thesis would have not been possible without the help and advice from Professor Ramon Costa Castelló; I really appreciate his support and patience during the realization of this project. Furthermore, I would also like to say thanks to all the ETSEIB's teachers, thanks their effort and being our guider during these years in the university. Finally, I want to take this opportunity to appreciate my parents, thanks for their encouragement and support throughout my life.

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Annex

Virtual laboratories designed using Easy Javascript Simulation.

- Two tanks system, web site and mobile devices versions.
- Pendulum, web site and mobile devices versions.